<u>The Naughtian Wave Equation</u>

Following is a derivation of the acoustic wave equation used for the mathematical modeling of waves generated by the collaborative efforts of Theta Naught and Alex Caldiero. Theta Naught itself could be represented as several variables with respect to the number of participants and/or instruments; however, for immediate purposes the denotation will be limited to a single variable, θ_0, to describe the flexible position of Theta Naught. Furthermore, the variable a_c will represent the unpredictable position of Alex Caldiero.

The general form of the wave equation can be defined by the partial differential equation

$$u_{tt} = c^2\left(u_{\theta\theta} + u_{aa}\right) \qquad \text{Eq. 1}$$

where the second derivative of wave velocity is equal to a constant times the sum of the second derivative of wave velocity with respect to Theta Naught and the second derivative of wave velocity with respect to Alex Caldiero, which can be expanded to

$$\frac{\partial^2 u(t,\theta_0,a_c)}{\partial t^2} = c^2\left[\frac{\partial^2 u(t,\theta_0,a_c)}{\partial \theta_0^2} + \frac{\partial^2 u(t,\theta_0,a_c)}{\partial a_c^2}\right] \qquad \text{Eq. 2}$$

where u is the wave velocity function dependent on time t, and the positions θ_0 and a_c. Also, c is the speed of sound (~343 meters per second). The second derivative of wave velocity is also known as wave *jerk*, wave *jolt*, or wave *shock*. So, if it is assumed that the wave velocity is zero at time-zero, then the initial condition in equation 3 is true. Furthermore, it can be assumed that the first derivative of wave velocity (or, wave acceleration) at time-zero is equal to a function, φ.

$$u(0,\theta_0,a_c) = 0 \qquad u(0,\theta_0,a_c) = \varphi(\theta_0,a_c) \qquad \text{Eq. 3 \& 4}$$

The solution to this partial differential equation to define the wave position can be found by a double integral over the unit sphere S defined by the wave acceleration function, φ.

$$u(t,\theta_0,a_c) = tM_{ct}[\varphi] = \frac{t}{4\pi}\iint_S \varphi(\theta_0 + ct\alpha, a_c + ct\beta)\,d\omega \qquad \text{Eq. 5}$$

where α and β are the first two coordinates on the unit sphere, $d\omega$ is the area element on the sphere, and ct is the radius. Simplifying and rewriting this equation over disc D yields

$$u(t,\theta_0,a_c) = \frac{1}{2\pi c}\iint_D \frac{\varphi(\theta_0+\xi, a_c+\eta)}{\sqrt{(ct)^2 - \xi^2 - \eta^2}}\,d\xi d\eta \qquad \text{Eq. 6}$$

The solution at (t, θ_0, a_c) depends not only on the data on the light cone where $(\theta_0-\xi)^2 + (a_c-\eta)^2 = c^2t^2$, but also on data that are interior to that cone.